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FUNDAMENTALS IN THE THEORY OF A WING PROFILE IN A SUPERSONIC FLOW

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CHAPTER 8

FUNDAMENTALS IN THE THEORY OF A WING PROFILE IN A SUPERSONIC FLOW

ABSTRACT. Velocity components are found for turbulence in supersonic flow at various Mach angles. Linear flow at obtuse angles close to 180° is stressed. Supersonic flow is studied around a plane plate. Flow conditions are considered which result in compression or rarefaction. Differences are noted in pressure distribution and lift along a plate in sub- and supersonic flow. Flow is compared in viscous and inviscid gases. Supersonic flow is examined around a thin profile with a sharp edge with respect to compression lines and rarefaction lines. Stress-strain diagrams are constructed for pressure distribution. The applicability of linear theory to a thin profile in subsonic flow is evaluated.

§ 8.1. The General Solution to a Linear Equation of Velocity Potential

In § 7.2 we obtained a linear equation of velocity profile for a plane flow /176* (7.8) which is applicable for both subsonic and supersonic flows. When $M_\infty > 1$ this equation is a hyperbolic type of linear equation (a wave equation) and therefore may not be reduced to a La Place equation with the aid of the real affine transform, as was done for subsonic flow (7.9).

Let us indicate the velocity potential ϕ in the following form:

$$\varphi = \varphi_\infty + \varphi', \tag{8.1}$$

where ϕ_∞ is the velocity potential for an undisturbed flow which at small angles of attack equals $v_\infty x + v_\infty \alpha y$;
ϕ' is the velocity potential for the turbulence.

Substituting expression (8.1) into equation (7.8), we obtain

$$(M_\infty^2 - 1)\frac{\partial^2 \varphi'}{\partial x^2} - \frac{\partial^2 \varphi'}{\partial y^2} = 0. \tag{8.2}$$

Let us introduce the new variables ξ and η according to the formulas

$$\left.\begin{aligned} \xi &= y - x \tan \mu_\infty \\ \eta &= y + x \tan \mu_\infty \end{aligned}\right\} \tag{8.3}$$

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* Numbers in the margin indicate pagination in the foreign text.

where μ_∞ is the Mach angle determined with respect to the velocity of the undisturbed flow

$$\tan \mu_\infty = \frac{1}{\sqrt{M_\infty^2 - 1}}.$$

Using the variables (8.3), equation (8.2) may be written in the following form /177

$$\frac{\partial^2 \varphi'}{\partial \xi \partial \eta} = 0. \tag{8.4}$$

A general solution to equation (8.4) is the function

$$\varphi' = f_1(\xi) + f_2(\eta). \tag{8.5}$$

This may be verified through direct substitution of the function in (8.5) into equation (8.4). The arbitrary functions f_1 and f_2 must be found from the boundary conditions of the concrete problem.

Returning to the previous variables x and y, we obtain

$$\varphi' = f_1(y - x|\tan \mu_\infty) + f_2(y + x|\tan \tag{8.6}$$

Let us find the components of the velocity of the turbulence

$$\left.\begin{aligned} v_x' &= \frac{\partial \varphi'}{\partial x} = (\dot{f}_2 - \dot{f}_1) \tan \mu_\infty \\ v_y' &= \frac{\partial \varphi'}{\partial y} = \dot{f}_1 + \dot{f}_2. \end{aligned}\right\} \tag{8.7}$$

where $\dot{f}_1$ and $\dot{f}_2$ are derivatives of the functions f_1 and f_2 according to the arguments $y \pm x \tan \mu_\infty$.

Here $y \pm x \tan \mu_\infty = \text{const}$ is the equation for the Mach lines in the first and second families in a nonturbulent flow.

Let us find the Mach angle in the turbulent flow

$$\sin \mu = \frac{1}{M}.$$

Substituting expression (7.5) instead of M, we obtain

$$\sin \mu = \sin \mu_\infty \frac{1}{1 + \frac{v_x'}{v_\infty}\left(1 + \frac{k-1}{2} M_\infty^2\right)}.$$

It follows that for $\frac{v'_x}{v_\infty} \ll 1$ within the limits of applicability of the linear theory, the Mach angle at any point in the flow may be taken as equal to the angle μ_∞: /178

$$\mu = \mu_\infty.$$

Under these conditions, the characteristics correspond to the Mach lines. Therefore, the family of straight lines $y \pm x\tan\mu_\infty = \text{const}$ is a rectilinear grid of the characteristics of the linear supersonic flow (Figure 8.1).

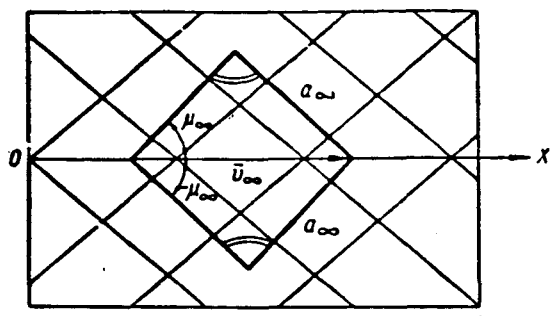


Figure 8.1. Rectilinear Grid of Characteristics for a Linear Supersonic Flow.

Let us find the projection of the vector for the velocity of the flow $\bar{v}$ in the direction of the Mach lines of the first and second families. From Figure 8.2 it follows that these projections equal

$$v_t = (v_\infty + v'_x)\cos\mu_\infty \pm v'_y \sin\mu_\infty,$$

where the minus sign corresponds to the Mach line of the first family and the plus sign corresponds to the Mach line of the second family.

Substituting expressions (8.7) here, we obtain

$$v_t = v_\infty \cos\mu_\infty - 2\dot{f}_1 \sin\mu_\infty.$$
$$v_t = a_\infty \cos\mu_\infty + 2\dot{f}_2 \sin\mu_\infty.$$

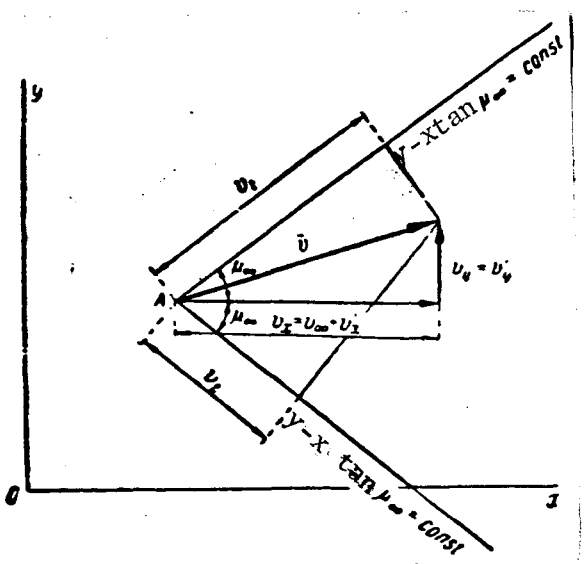


Figure 8.2. Determining the Components for the Velocity of the Flow along the Mach Lines.

Along the corresponding Mach lines, $\dot{f}_1$ and $\dot{f}_2$ are constant. Therefore the projections of v_t along the Mach line of the first and second families retain a constant value.

§ 8.2. Linear Supersonic Flow Around Obtuse Angles. /179

As an example let us observe the linear supersonic flow around obtuse angles close to 180° (Figure 8.3). The angle $\Delta\theta$ will be considered positive for an angle greater than 180° (Figure 8.3a) and negative for angles less than 180° (Figure 8.3b).

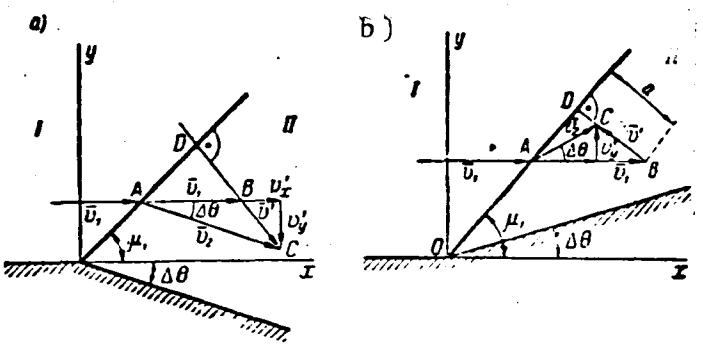


Figure 8.3. Linear Supersonic Flow Around Obtuse Angles.

Extending the Mach line $y = x\tan\mu_1$, we obtain the regions of nonturbulent (I) and turbulent (II) flows. In region I the velocity is constant everywhere and equal to $\bar{v} = \bar{v}_1$, while in region II, the velocity vector $\bar{v}_2$ forms the angle $\Delta\theta$ with the vector $\bar{v}_1$. Let us write the boundary conditions for the problem.

1. In region I the nonturbulent flow $y \geq x \tan \mu_1$, $v'_x = v'_y = 0$, $\varphi' = 0$.

2. From Figure 8.3 it follows that in region II $(-x\Delta\theta \leqslant y < x \tan \mu_1)$ fulfills the following conditions

$$-\frac{v'_y}{v_x} = \Delta\theta,$$

or

$$-\frac{v'_y}{v_1 + v'_x} = \Delta\theta.$$

Whence within the limits of linear theory, discarding the value $v'_x\Delta\theta$ of the second order of smallness, we obtain

$$v'_y = -v_1\Delta\theta. \tag{8.8}$$

In addition, for the problem under consideration the function $f_2 = 0$ in all the planes, because the rotation occurs on the Mach line $y - x \tan \mu_1 = 0$, while the Mach line for the other family is outside the flow. Substituting the boundary conditions for $y \geq x \tan \mu_1$ into equation (8.6), we obtain $f_1 = 0$ when $y \geq x \tan \mu_1$. From formula (8.7) it follows that in the given case /180

$$v'_x = -\dot{f}_1 \tan \mu_1, \quad v'_y = \dot{f}_1.$$

Here μ_1 is the Mach angle corresponding to the velocity of the nonturbulent flow v_1.

Replacing v'_y by $\dot{f}_1$ under the conditions in (8.8), we obtain the usual

differential equation for determining f_1:

$$\dot{f}_1 = -v_1 \Delta\theta.$$

Whence

$$f_1 = -v_1 (y - x \tan \mu_1)\Delta\theta + C,$$

where C is the integration constant.

Considering that $f_1 = 0$ when $y = x \tan \mu_1$, we find that $C = 0$. Then

$$\varphi' = f_1 = -v_1(y - x \tan \mu_1) \Delta\theta. \tag{8.9}$$

Let us find the constituent Mach velocities

$$v'_x = \frac{\partial \varphi'}{\partial x} = v_1 \tan \mu_1 \Delta\theta,$$
$$v'_y = \frac{\partial \varphi'}{\partial y} = -v_1 \Delta\theta,$$

or

$$\left.\begin{aligned} v'_x &= \frac{\partial \varphi'}{\partial x} = \frac{v_1 \Delta\theta}{\sqrt{M_1^2 - 1}} \\ v'_y &= \frac{\partial \varphi'}{\partial y} = -v_1 \Delta\theta \end{aligned}\right\}. \tag{8.10}$$

From formula (8.10) it follows that if $\Delta\theta > 0$ (an obtuse angle greater than 180°), $v'_x > 0$ while $v'_y < 0$. This being the case, $v_2 > v_1$, i.e. there is a rarified flow. If $\Delta\theta < 0$ (an obtuse angle less than 180°), then $v'_x < 0$, while $v'_y > 0$, which corresponds to compressed flow ($v_2 < v_1$).

Using expressions (8.10), we find the ratio between the components of the Mach velocity

$$\frac{v'_x}{-v'_y} = \tan \mu_1.$$

Then from Figure 8.3 it follows that the vector of the Mach velocity $\bar{v}'_1$ is perpendicular to the Mach line for flow around obtuse angles. This property of the vector $\bar{v}'$ permits us to easily find the vector for the velocity of the turbulent flow $\bar{v}_2$ according to the given vector $\bar{v}_1$. To do so, we need only construct the triangle ABC (see Figure 8.3) in which the side AB = v_1, the angle at the apex A equals the given angle $\Delta\theta$ and the end of the velocity vector $\bar{v}_2$ (point C) lies on the straight line running from point B perpendicular to the Mach line. /181

The perpendicularity of the vector $\bar{v}'$ to the Mach line results from the theorem of variations in the amounts of motion. On the basis of this theorem, projection of the velocity vector along the Mach line does not change as the flow changes its angle. This indicates that $\bar{v}'$ is perpendicular to the Mach line.

Let us find the change in pressure. To do so, let us use the linear Bernoulli equation (7.6):

$$p_2 = p_1 - \rho_1 v_1 v'_x.$$

Substituting the expression v'_x into (8.10) we obtain

$$p_2 = p_1 - \frac{2q_1 \Delta\theta}{\sqrt{M_1^2 - 1}}, \tag{8.11}$$

where q_1 is the velocity head of the nonturbulent flow equal to $\frac{\rho_1 v_1^2}{2}$.

Whence we find the pressure coefficient

$$\bar{p} = \frac{p_2 - p_1}{q_1} = -\frac{2\Delta\theta}{\sqrt{M_1^2 - 1}}. \tag{8.12}$$

From formula (8.11) it follows that for flow around an angle greater than 180° ($\Delta\theta > 0$), pressure decreases while for an angle less than 180° ($\Delta\theta < 0$), pressure increases. As a result of this, for definiteness we will call the Mach line the rarefaction line when $\Delta\theta > 0$ and the compression line when $\Delta\theta < 0$.

It is well known that flow around an obtuse angle less than 180° results in sharp drops in compression (Chapter V). The compression line is an infinitely slight drop in density with the angle of inclination $\beta = \mu_1$. For flow around an angle greater than 180°, there is a continuous rarefaction flow and a beam of rarefaction lines is formed (§ 6.8). Consequently, in linear theory the drop in the finite intensity is replaced by a drop in infinitely small intensity ($\beta = \mu_1$), while the bundle of rarefaction lines is replaced by a single rarefaction line according to the velocity $\bar{v}_1$. Therefore, linear theory yields a decrease of values in pressure due to the compression drop and increased rarefaction for flow around angles greater than 180°. /182

§ 8.3. The Linear Theory of Supersonic Flow Around a Plane Plate.

The diagram from § 8.2 concerning rarefied flow and compressed flow permits us to solve rather easily the problem of supersonic flow around a plane plate at a small angle of attack (Figure 8.4).

In supersonic flow, the small turbulences against the velocity direction

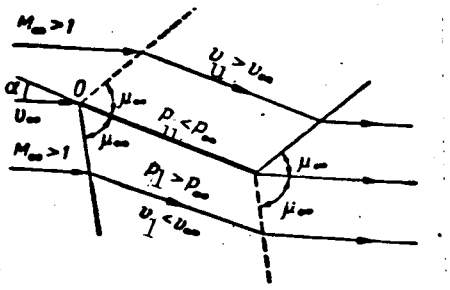


Figure 8.4. Linear Theory on Supersonic Flow Around a Plane Plate.

are not extended. Therefore, a nonturbulent flow strikes the plane plate. For this reason, flow around the upper and lower surfaces at $M_\infty > 1$ may be considered independent from each other. Then the flow around the upper surface corresponds to flow around an angle greater than 180° with the angle of turn for the flow equal to + α while flow around the lower surface is equivalent to flow around an angle less than 180° (the angle of turns for the flow being equal to - α). In this case, a rarefied flow forms on the leading edge on the side of the upper surface, while a compressed flow forms on the lower surface.

Because there are no sources of turbulence between the leading and trailing edges of either the upper or lower surfaces, the velocities of the flow and the pressure on these surfaces are constant and equal to v_u and p_u for the upper surface and v_l and p_l for the lower surface.

On the trailing edge, when the flow moves around the upper surface it turns at an angle - α, which results in the formation of a compressed line (there is compressed flow), while on the lower surface the angle of turn for the flow equals + α, which results in a rarefied line (there is rarefied flow).

To determine the pressure and the pressure coefficients on the surface of the plane plate, we may use formulas (8.11) and (8.12), substituting into them /183 $M_l = M_\infty$, $p_l = p_\infty$ and for the points on the upper surface $\Delta\theta = \alpha$, while for the lower surface $\Delta\theta = -\alpha$. Then

$$\left.\begin{aligned} p_u &= p_\infty - \frac{2q_\infty \alpha}{\sqrt{M_\infty^2 - 1}} \\ p_l &= p_\infty + \frac{2q_\infty \alpha}{\sqrt{M_\infty^2 - 1}} \end{aligned}\right\} \qquad (8.13)$$

and

$$\left.\begin{aligned} \bar{p}_u &= -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \\ \bar{p}_l &= \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} \end{aligned}\right\}. \qquad (8.14)$$

Let us note several features concerning supersonic flow around a plate.

From Figure 8.5 it follows that in supersonic flow, as opposed to subsonic flow, the pressure along the plate is distributed uniformly. Therefore, the center of pressure is distributed along the center of the plate (x_{sub} = 1/2 u or $\bar{x}_{sub}$ = 0.5). In subsonic flow, as is well known, $\bar{x}_{sub}$ = 0.25. In addition, according to the results of linear theory (8.14), $\bar{p}_1 = |\bar{p}_u|$. This means that in supersonic flow, lift is created equally on both the upper and lower surfaces. In subsonic flow, however, approximately three quarters of the lift results from the effect of rarefaction on the upper surface and only one quarter of the lift is caused by pressure on the lower surface.

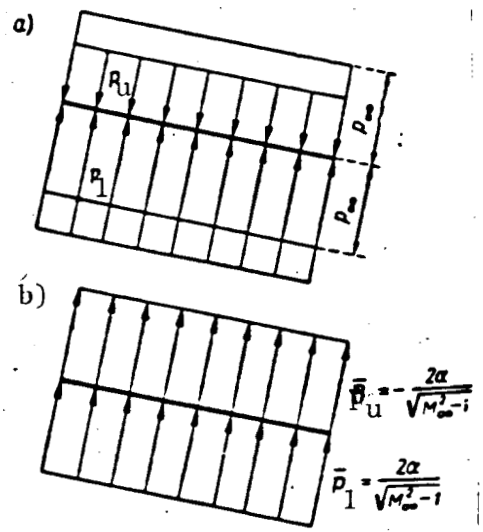


Figure 8.5. Stress-Strain Diagram of Pressure Distribution (a) and Pressure Coefficient (b) along a Plate.

Let us calculate the full aerodynamic force acting on a unit of length of the span of a plate:

$$R = (\bar{p}_1 - \bar{p}_u) q_\infty b \cdot 1.$$

Projecting the vector of the equivalent aerodynamic force R on the normal to the direction of velocity of the incident flow and along the velocity $\bar{v}_\infty$ (Figure 8.6), we obtain the lifting force Y and the force of wave resistance X: /184

$$Y = R\cos\alpha.$$
$$X = R\sin\alpha.$$

For small angles of attack, changing cos α = 1, while sin α = α, we obtain

$$Y = R,$$
$$X = Y\alpha.$$

Whence

$$\left.\begin{aligned} c_y &= \bar{p}_1 - \bar{p}_u \\ c_x &= c_y\alpha \end{aligned}\right\} \quad (8.15)$$

Substituting expressions (8.14) into formulae (8.15), we obtain

$$c_y = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}, \quad (8.16)$$

$$c_x = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}, \quad (8.17)$$

where α is the angle of attack in radians.

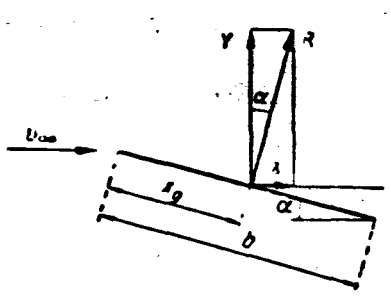


Figure 8.6. Determining the Lift Force and Wave Resistance of a Plane Plate.

For small angles of attack (in the linear section of the curve for the dependence of c_y on α), the coefficient for the lifting force may be presented in the following form:

$$c_y = c_y^{\alpha^\circ} \alpha^\circ. \tag{8.18}$$

From formula (8.16) it follows that

$$c_y^{\alpha^\circ} = \frac{4}{57{,}3\sqrt{M_\infty^2 - 1}}. \tag{8.19}$$

The value $c_y^{\alpha^\circ}$ in supersonic flow is substantially less than in subsonic flow, as a result of which the greater M_∞ is, the smaller $c_y^{\alpha^\circ}$ will be (Figure 8.7). For example, in an incompressible flow (M = 0), according to the theory of thin profiles, $c_y^{\alpha^\circ} = \frac{2\pi}{57.3} = 0.1096$, while for $M_\infty = 2$ $c_y^{\alpha^\circ} = 0.04$ and for $M_\infty = 4$ $c_y^{\alpha^\circ} = 0.018$.

We obtain the lifting force as a function of M_∞:

$$Y = c_y \frac{\rho_\infty v_\infty^2}{2} S.$$

Let us present the velocity head in the following form /185

$$\frac{\rho_\infty v_\infty^2}{2} = \frac{k}{2} p_\infty M_\infty^2.$$

Then, in supersonic flow the lift will be proportional to the $\frac{M_\infty^2}{\sqrt{M_\infty^2 - 1}}$:

$$Y = 2\kappa\alpha p_\infty S \frac{M_\infty^2}{\sqrt{M_\infty^2 - 1}},$$

when in the incompressible flow the lift increases according to the parabolic law $(Y = \pi\kappa\alpha S p_\infty M_\infty^2)$ as the Mach number M_∞ increases.

In Formula (8.17), c_x is the coefficient for the wave resistance. For continuous flow of an inviscid gas around a plane plate ($\lambda = \infty$), there are no

other types of resistance. In the general case, for the flow of a viscous gas the coefficient of resistance equals

$$c_x = 2c_f + \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}. \qquad (8.20)$$

where c_f is the resistance coefficient for friction of a plane plate, which is a function of the flow conditions for the viscous gas in the boundary layer, of the Re number and M_∞ (see Chapter XII).

Let us find the moment of the aerodynamic force relative to the leading edge. From Figure 8.6 it is obvious that

$$M = - R \frac{b}{2}.$$

Here the moment during pitching is considered positive.

For small angles of attack, replacing R by Y, we obtain

$$M = - Y \frac{b}{2}.$$

Then the coefficient for the moment will equal

$$c_m = -\frac{1}{2} c_y = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}. \qquad (8.21)$$

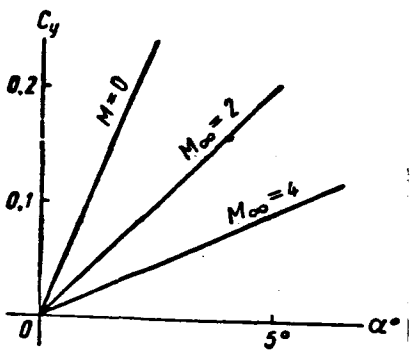


Figure 8.7. The Coefficient of Lift as a Function of the Angle of Attack for Various Values of the Mach Number M_∞.

Using the formulas (8.16) and (8.20), let us find the aerodynamic quality for the plate /186

$$K = \frac{c_y}{c_x} = \frac{4\alpha}{2c_f \sqrt{M_\infty^2 - 1} + 4\alpha^2}. \qquad (8.22)$$

Without taking into consideration the friction drag coefficient, the aerodynamic quality equals

$$K = \frac{1}{\alpha}.$$

The greater the angle of attack is, the lower the quality of the plate.

§ 8.4. Linear Theory of a Thin Profile in a Supersonic Flow.

Let us examine supersonic flow around a thin profile with a sharp edge. This problem may be reduced to that of successive flow at obtuse angles, if

the smooth contour of the profile is replaced by a broken line (Figure 8.8). Then, depending on the thickness of the profile and the angle of attack, there may be various cases of flow as shown in Figure 8.9. For small angles of attack (Figure 8.9a) compression waves appear on the leading and trailing edges of both the lower and upper surfaces. If the angle of attack is greater than the angle of incidence of the tangent to the surface at points A and B, there is first compressed flow on the leading edge on the side of the lower surface, while a rarefaction wave appears on the side of the upper surface; on the trailing edge there /187
are a compression wave on the upper surface and a rarefaction wave on the lower surface. Between the leading and trailing edges on both the lower and upper surfaces there is rarefied flow. Therefore, a rarefaction wave leaves each point on the surface.

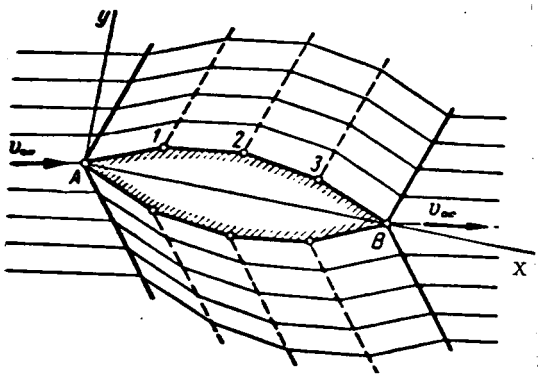


Figure 8.8. A Linear Supersonic Flow Around a Profile.

Let us propose that the upper and lower surfaces are given by the equations

$$y_u = y_u(x),$$
$$y_e = y_e(x).$$

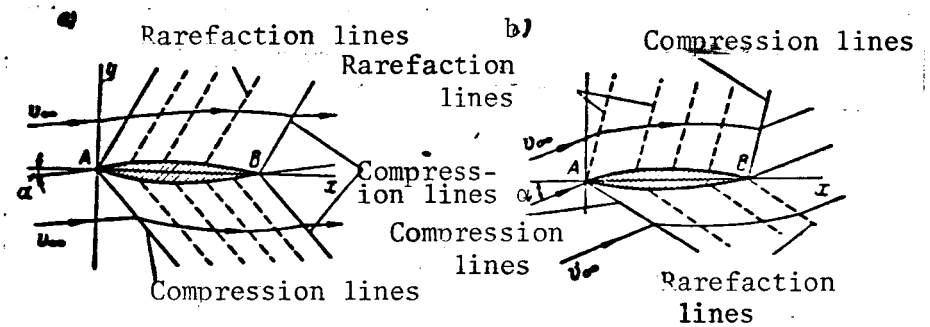


Figure 8.9. Various Cases of Linear Supersonic Flow Around a Thin Profile.

Let us find the pressure on the profile surface. To determine the pressure, let us use formula (8.11).

In sector A1 (see Figure 8.8) the angle of turn for the flow equals

$$\alpha - \theta'_0,$$

$$\alpha - \frac{dy_u}{dx}; \tag{8.26}$$

while the pressure for the points on the upper surface equals

$$p_u = p_\infty - \frac{2q_\infty\left(\alpha - \frac{dy_u}{dx}\right)}{\sqrt{M_\infty^2 - 1}}. \tag{8.27}$$

The pressure coefficient is

$$\bar{p}_u = -\frac{2\left(\alpha - \frac{dy_u}{dx}\right)}{\sqrt{M_\infty^2 - 1}}. \tag{8.28}$$

Similarly, using formulas (8.11) and (8.12), we may obtain the formula for determining the pressure coefficient at points on the lower surface

$$\bar{p}_l = \frac{2\left(\alpha - \frac{dy_l}{dx}\right)}{\sqrt{M_\infty^2 - 1}}. \tag{8.29}$$

From formulas (8.28) and (8.29) it follows that the pressure coefficient at any point on the surface of the profile for a given Mach value M_∞ is a /189

function of only the local angle of attack $\left(\alpha - \frac{dy_u}{dx}\right)$ and and not

of the inclination of the other surface elements.

In addition, the pressure coefficient may be represented in the form of the following sum

$$\bar{p} = \bar{p}^* + \bar{p}^{**}, \tag{8.30}$$

where

$$\bar{p}^* = \pm \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}, \tag{8.31}$$

for points on the upper surface $\bar{p}^{**} = \frac{2\frac{dy_u}{dx}}{\sqrt{M_\infty^2 - 1}}$, and for points on the lower surface $\bar{p}^{**} = -\frac{2\frac{dy_l}{dx}}{\sqrt{M_\infty^2 - 1}}$.

where θ'_0 is the angle of inclination of the element A1 to the chord, equal to $\left(\frac{dy_u}{dx}\right)_{x=0}$.

If $\alpha > \theta'_0$ ($\alpha - \theta'_0 > 0$), then there is rarefied flow (Figure 8.9b), while if $\alpha < \theta'_0$ ($\alpha - \theta'_0 < 0$), there is compressed flow (8.9a). The pressure on the segment A1 is

$$p_1 = p_\infty - \frac{2q_\infty(\alpha - \theta'_0)}{\sqrt{M_\infty^2 - 1}}. \tag{8.23}$$

Going from the surface element A1 to the element 1, 2, the flow turns at an angle $\theta'_0 - \theta'_1$. Here $\theta'_1 = (dy_u/dx)_1$. When this is true, there is rarefied flow.

Therefore

$$p_2 = p_1 - \frac{2q(\theta'_0 - \theta'_1)}{\sqrt{M_1^2 - 1}},$$

where q_1 is the velocity head; /188

M_1 is the Mach number on the segment A1.

Within the limits of applicability of linear theory, we may assume

$$\frac{q_1}{\sqrt{M_1^2 - 1}} = \frac{q_\infty}{\sqrt{M_\infty^2 - 1}}.$$

Then

$$p_2 = p_1 - \frac{2q_\infty(\theta'_0 - \theta'_1)}{\sqrt{M_\infty^2 - 1}}. \tag{8.24}$$

Substituting expression (8.23) into formula (8.24), we obtain

$$p_2 = p_\infty - \frac{2q_\infty(\alpha - \theta'_1)}{\sqrt{M_\infty^2 - 1}}. \tag{8.25}$$

In formulas (8.23) and (8.25) the angles $\alpha = \theta'_0$ and $\alpha = \theta'_1$ are local angles of attack. The local angle of attack is the angle between the surface element and the velocity vector for a nonturbulent flow. At any point on the upper surface, the local angle of attack may be represented in the form

For symmetrical profiles ($y_l = -y_u$ and $\frac{dy_l}{dx} = -\frac{dy_u}{dx}$), the pressure coefficient $\bar{p}^{**}$ equals

$$\bar{p}^{**} = \frac{2\frac{dy_u}{dx}}{\sqrt{M_\infty^2 - 1}}. \qquad (8.32)$$

From formulas (8.31) and (8.32) it follows that $\bar{p}^*$ is the pressure coefficient on the surface of a profile with zero thickness (a plane plate) at the given angle of attack, while $\bar{p}^{**}$ is the pressure coefficient on a surface of a given profile for a zero angle of attack. Consequently, within the limits of applicability of linear theory, the effect of the angle of attack and the thickness of the profile on the value $\bar{p}$ may be studied independently of each other (the effect of the angle of attack in the case of flow around a plane plate, the effect of thickness and the shape of the profile for $\alpha = 0$). Then the overall stress-strain diagram for the distribution of pressure along the profile surface may be obtained as a result of superimposing the two diagrams -- the one for pressure distribution along the surface of the plane plate for $\alpha \neq 0$ (see Figure 8.5) and that for pressure along the profile surface for $\alpha = 0$.

Let us examine some examples of pressure distribution along the surface of symmetrical profiles for $\alpha = 0$. Because under these conditions the pressure at symmetrical points on the profile is identical, when we have a zero angle of attack, the curves for pressure distribution along the upper and lower surfaces correspond. To construct a stress-strain diagram for the distribution of the pressure coefficient, let us lay out the coefficients $\bar{p}$ along the normal to the chord in which the positive values run downward from the chord and the negative values run upward.

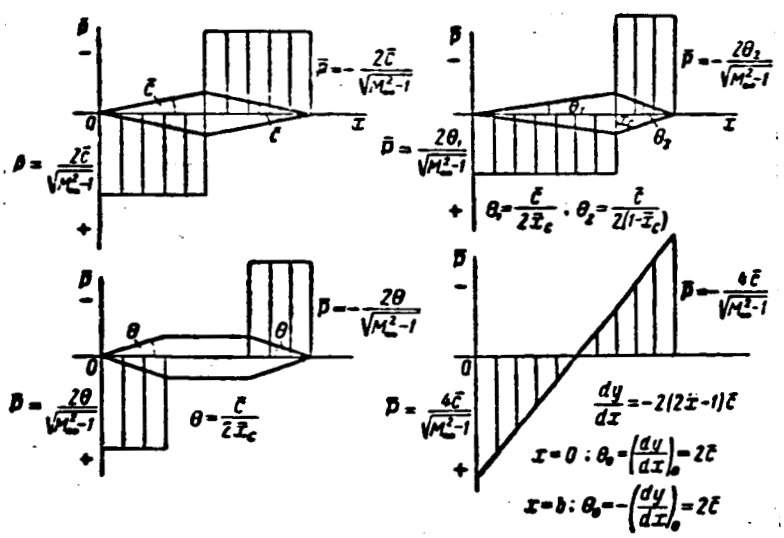


Figure 8.10. Stress-Strain Diagram for the Pressure Distribution along the Surface of Profiles According to Linear Theory ($\alpha = 0$).

As an example, Figure 8.10 shows the stress-strain diagrams for the distribution of the pressure coefficients along the surface of a Rhomboid, rectangular, hexagonal and lenticular (formed by two arcs of a circle) profiles for $\alpha = 0$. The pressure coefficient for the given value of the Mach number M_∞ may be calculated according to formula (8.32). /190

Let us find the lift, the resistance and the moment of forces relative to the leading edge of the profile.

The elementary lift acting on the element of the lower and upper surfaces (Figure 8.11) equals

$$dY_1 = p_1 \cos\theta_1 \, ds_1 \, l,$$
$$dY_u = -p_u \cos\theta_u \, ds_u \, l,$$

where θ is the local angle of attack equal to $\alpha - \frac{dy}{dx}$;

ds is the length of the elementary arc along the surface contour;

l is the length of the wing segment along the span.

The subscripts 1 and u indicate the values relating correspondingly to the lower and upper surfaces.

For thin profiles at a small angle of attack, we may use

$$ds_1 = ds_u = dx, \quad \cos\theta_1 = 1 \quad \text{and} \quad \cos\theta_u = 1.$$

Then /191

$$dY = dY_1 + dY_u = (p_1 - p_u) \, l \, dx,$$

or

$$dY = (\bar{p}_1 - \bar{p}_u) \, q_\infty \, dx.$$

The total lift is

$$Y = q_\infty l \int_0^b (\bar{p}_1 - \bar{p}_u) \, dx.$$

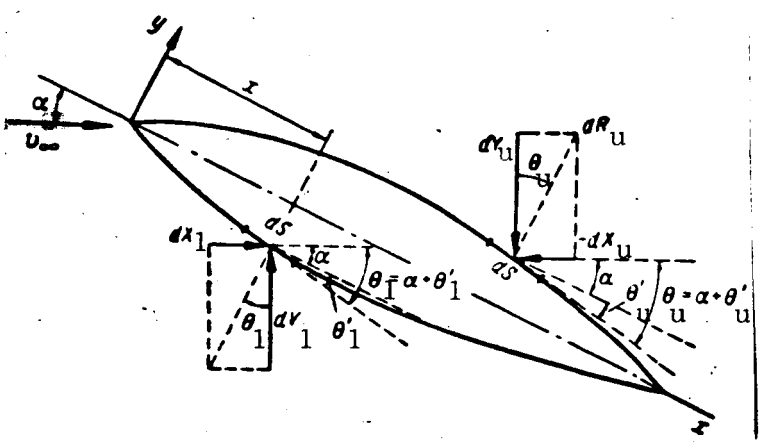


Figure 8.11. Calculating the Aerodynamic Forces Acting on the Profile in Supersonic Flow.

Substituting the expressions (8.28) and (8.29) instead of $\bar{p}_1$ and $\bar{p}_u$ and considering that $\int_0^b \frac{dy_1}{dx} dx = \int_0^b \frac{dy_u}{dx} dx = 0$, we obtain

$$Y = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} q_\infty l b,$$

where b is the chord of the profile.

Whence

$$c_y = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}. \tag{8.33}$$

From formulas (8.33) it follows that within the limits of applicability of linear theory, the coefficient for the lift is not a function of the shape and thickness of the profile and, all other things equal, corresponds to the coefficient of the lift for a plane plate (8.16). /192

Let us proceed to determining the resistance. The elementary resistances acting on the elements of the lower and upper surfaces $l ds_l = l dx$ and $l ds_u = l dx$ equals

$$dX_l = p_l \sin\theta_l \, l ds_l,$$
$$dX_u = -p_u \sin\theta_u \, l ds_u,$$

where for small local angles of attack θ_l and θ_u we may take

$$\sin\theta_l = \theta_l = \alpha - \frac{dy_l}{dx},$$
$$\sin\theta_u = \theta_u = \alpha - \frac{dy_u}{dx}.$$

Then

$$dX = dX_l + dX_u = (\bar{p}_l \theta_l - \bar{p}_u \theta_u) q_\infty l dx.$$

Substituting expressions (8.28) and (8.29) instead of $\bar{p}_l$ and $\bar{p}_u$ and integrating, we obtain

$$X = \left(\frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{B}{\sqrt{M_\infty^2 - 1}} \right) q_\infty l b.$$

Whence

$$c_x = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{B}{\sqrt{M_\infty^2 - 1}}. \tag{8.34}$$

Here

$$B = \frac{2}{b} \int_0^b \left[\left(\frac{dy_l}{dx} \right)^2 + \left(\frac{dy_u}{dx} \right)^2 \right] dx. \tag{8.35}$$

Let us transform expression (8.35). To do so, the ordinates of the profile are set in the following form

$$y_1 = c\bar{y}_1(x) \quad \text{and} \quad y_u = c\bar{y}_u(x),$$

where c is the profile thickness, while $\bar{y} = \frac{y}{c}$.

Let us proceed to the relative coordinates along the chord:

$$\bar{x} = \frac{x}{b}.$$

Then

$$\frac{dy_u}{dx} = \bar{c}\frac{d\bar{y}_u}{d\bar{x}}, \text{ and } \frac{dy_1}{dx} = \bar{c}\frac{d\bar{y}_1}{d\bar{x}}.$$

Substituting these expressions into formula (8.35), we have /193

$$B = 2\bar{c}^2\int_0^1\left[\left(\frac{d\bar{y}_1}{d\bar{x}}\right)^2 + \left(\frac{d\bar{y}_u}{d\bar{x}}\right)^2\right]d\bar{x}.$$

Let us introduce the following designations:

$$2\int_0^1\left[\left(\frac{d\bar{y}_1}{d\bar{x}}\right)^2 + \left(\frac{d\bar{y}_1}{d\bar{x}}\right)^2\right]d\bar{x} = K_1,$$

where K_1 is a function of only the shape of the profile.

Then

$$B = K_1\bar{c}^2.$$

Substituting expression B into formula (8.34), we obtain

$$c_x = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} + \frac{K_1\bar{c}^2}{\sqrt{M_\infty^2 - 1}}. \tag{8.36}$$

From formula (8.36) it follows that the coefficient for the wave resistance for the profile may be presented in the form of the sum of the two resistance coefficients:

1. The minimum coefficient for the wave resistance (for $\alpha = 0$) equal to

$$c_{xu} = \frac{K_1\bar{c}^2}{\sqrt{M_\infty^2 - 1}}, \tag{8.37}$$

which for the given value of M_∞ is a function of only the shape of the profile and the relative thickness $\bar{c}$.

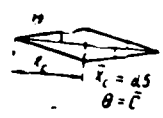
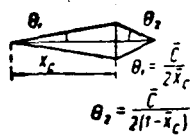
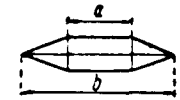
2. The coefficient of wave resistance for a profile of zero thickness ($\bar{c} = 0$) for the given angle of attack

$$c_{xi} = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}},$$

which corresponds to the coefficient of wave resistance for a plane plate (8.17). This resistance is usually called inductive wave resistance, inasmuch as it arises only for $c_y \neq 0$ and is indicated by c_{xi}.

Table 8.1 shows the values for the coefficients K_1 for profiles of various shapes. From the table it follows that the least wave resistance, which all other things equal, belongs to the Rhomboid profile for which the coefficient $K_1 = 4$. As an example, Figure 8.12 shows the curve for the resistance of the coefficient of minimum wave resistance (8.37) for the Rhomboid profile on the relative thickness for $M_\infty = 3$. /194

TABLE 8.1

Profile	Rhomboid	Rectangular	Hexagonal	Formed by arcs of a circle
				
K_1	4	$\frac{1}{\bar{x}_c(1-\bar{x}_c)}$	$\frac{4}{1-\frac{a}{b}}$	$\frac{16}{3}$

Let us write the expression for the moment of a profile relative to the leading edge. It equals

$$M_0 = \int_0^b (\bar{p}_1 - \bar{p}_u) q_\infty x dx.$$

Substituting here the expressions $\bar{p}_u$ and $\bar{p}_1$ according to formulas (8.28) and (8.29), as a result of integrating we obtain

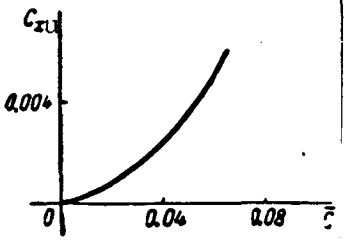


Figure 8.12. The Value c_{xu} as a Function of the Relative Thickness $\bar{c}$.

$$c_m = \frac{M_z}{\frac{\rho_\infty v_\infty^2}{2} Sb} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}} + c_{m_0},$$

where c_m is the coefficient of the moment for the profile relative to the leading edge;
S is the area equal to b·1;
c_{m_0} is the coefficient of the moment at a zero angle of attack equal to

$$c_{m_0} = \frac{2\bar{c}}{\sqrt{M_\infty^2 - 1}} \int_0^1 \left(\frac{d\bar{y}_l}{d\bar{x}} + \frac{d\bar{y}_u}{d\bar{x}} \right) \bar{x} d\bar{x}.$$

For a symmetrical profile for any value

$$\frac{d\bar{y}_l}{d\bar{x}} = -\frac{d\bar{y}_u}{d\bar{x}}$$

Therefore

$$c_{m_0} = 0 \text{ and } c_m = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}.$$

Knowing the values for the coefficients c_m and c_y, we may determine the position of the aerodynamic center which is a point relative to which the moment of the aerodynamic forces is not a function of the angle of attack. To do so, let us write the expression for the moment M(x) relative to an arbitrary point on the profile. On the basis of the theorem of moments, it is equal to /195

$$M(x) = M_0 + Yx$$

and

$$c_m(x) = c_m + c_y \bar{x}.$$

Substituting here the expressions for c_m and c_y, we obtain

$$c_m(x) = c_{m_0} - \frac{2\alpha}{\sqrt{M_\infty^2 - 1}} + \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} \bar{x}.$$

If the point under consideration corresponds to the aerodynamic center

($x = x_F$), the coefficient of the moment is not a function of α and is equal to $c_m = c_{m_0}$. Therefore

$$-\frac{2\alpha}{\sqrt{M_\infty^2-1}}+\frac{4\alpha}{\sqrt{M_\infty^2-1}}\bar{x}_F=0.$$

Whence $\bar{x}_F = \frac{1}{2}$.

Consequently, the center of any thin profile in a supersonic flow is located in the middle of the chord. As it is well known, in subsonic flow the center of a thin profile is one quarter of a chord distant from the leading edge.

Let us note the limits of applicability of the linear theory. Linear theory yields relatively good results within the following range of Mach numbers M_∞:

$$1,2<M_\infty<5.$$

In the transonic range, where $1 < M < 1.2$, linear theory is inapplicable because with a relatively low supersonic velocity in the incident flow, even for flow around thin bodies, we encounter detached compression shocks beyond whose central section the flow becomes subsonic. For hypersonic velocities ($M > 5$), linear theory is also incorrect because in this case the basic assumptions of linearization may not be fulfilled. This is explained by the fact that in a hypersonic flow the small relative change in velocity leads to a large change in pressure, density, temperature, speed of sound, etc., (see Chapter XI).

As was noted in Chapter VII, linearization is applied in a subsonic subcritical flow ($M_\infty < M_{cr}$). In the transonic range $M_{cr} < M_\infty < 1$, the method /196
of linearization is also inapplicable because in this range of Mach numbers M_∞ with the incident flow traveling at subsonic velocity, there appear zones of local supersonic velocities around the wing. Consequently, in the transonic range $M_{cr} < M < 1.2$ we encounter mixed flow around the profile: when the incident flow is traveling at a subsonic velocity, local supersonic velocities are formed ($M_{cr} < M_\infty < 1$) or when the incident flow is traveling /197
at a supersonic velocity, a zone of subsonic velocities is created beyond the detached shock.

In conclusion, let us introduce the character of the coefficients c_y and c_x of the thin profile as a function of the Mach number M_∞ (Figures 8.13 and 8.14). These dependences are constructed according to the results of linear theory. In addition, the figures show the representative character of the dependence of c_y and c_x on M_∞ in the transonic range. Figure 8.14 shows the

dependence of c_x with zero lift.

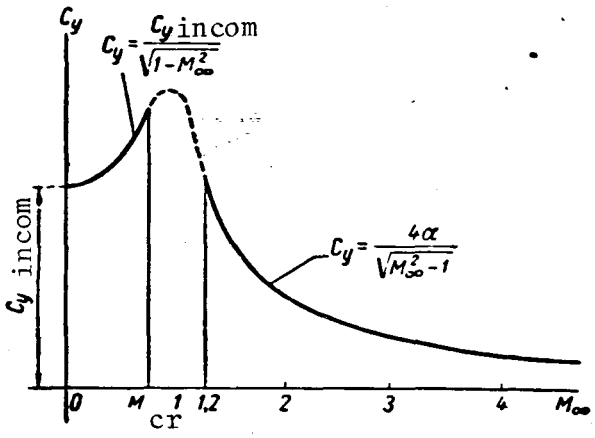


Figure 8.13. Curve of the Lift Coefficient as a Function of the Mach Number M_∞.

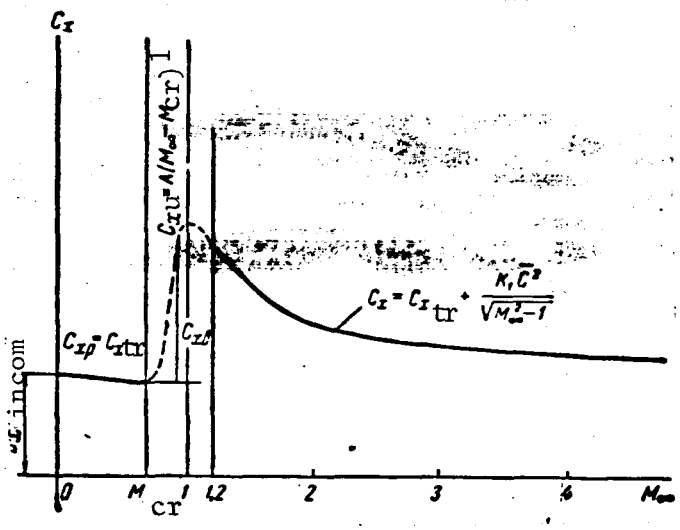


Figure 8.14. Curve of the Profile Resistance Coefficient as a Function of the Mach Number M_∞ for $c_y = 0$.

§ 8.5. Refined Theories of a Profile in a Supersonic Flow.

Earlier we examined the theory of the first approximation in which the dependence between the pressure difference and the angle of incidence of the flow on a nonturbulent direction was taken as linear

$$p = p_\infty \pm \frac{2\theta}{\sqrt{M_\infty^2 - 1}} q_\infty = p_\infty \pm c_1 q_\infty \theta,$$

where q_∞ and M_∞ are the velocity head and the Mach number for a nonturbulent flow.

This dependence is correct for relatively small values of the angle θ, i.e. for quite thin profiles with the flow at a small angle of attack.

Let us examine the basic assumptions of the theory of the second approximation. Using expansion to a Taylor series and being restricted by only the first two terms of the expansion, we obtain the dependence between the pressure difference Δp and the angle of rotation of flow θ in the following form

$$p = p_\infty + (c_1 \theta + c_2 \theta^2) q_\infty. \tag{8.38}$$

Then

$$\left.\begin{aligned} p_1 &= p_\infty + (c_1\theta_1 + c_2\theta_1^2)\, q_\infty \\ p_u &= p_\infty + (-c_1\theta_u + c_2\theta_u^2)\, q_\infty \end{aligned}\right\}. \tag{8.39}$$

The subscripts 1 and u, as previously, refer to the corresponding lower and upper surfaces of the profile. The coefficients c_1 and c_2 equal

$$c_1 = \frac{2}{\sqrt{M_\infty^2 - 1}}, \quad c_2 = \frac{1,2\,M_\infty^4 - 2M_\infty^2 + 2}{(M_\infty^2 - 1)^2}. \tag{8.40}$$

For lift Y, head resistance X and the moment M the following formulas are correct /198

$$Y = \int_0^{s_1} \Delta p_1 \cos\theta_1\, ds_1 - \int_0^{s_u} \Delta p_u \cos\theta_u\, ds_u \tag{8.41}$$

$$X = \int_0^{s_1} \Delta p_1 \sin\theta_1\, ds_1 - \int_0^{s_u} \Delta p_u \sin\theta_u\, ds_u \tag{8.42}$$

$$M_z = \int_0^{s_1} \Delta p_1 \cos\theta_1\, x\, ds_1 - \int_0^{s_u} \Delta p_u\, x \cos\theta_u\, ds_u. \tag{8.43}$$

The integrals (8.41) to (8.43) may be calculated if the geometric characteristics of the profile are known.

Let us substitute the values of Δp_1 and Δp_u (8.39) into the expression for Y (8.41) and, as we did earlier, let us calculate approximately

$$\cos\theta \cong 1, \quad \sin\theta \cong \theta, \quad s_1 = s_u = b.$$

As a result we obtain

$$Y = \int_0^b (c_1\theta_1 + c_2\theta_1^2)\, q_\infty\, dx - \int_0^b (-c_1\theta_u + c_2\theta_u^2)\, q_\infty\, dx.$$

Introducing the new variable $\bar{x} = \frac{x}{b}$, we will have

$$c_y = \int_0^1 (c_1\theta_1 + c_2\theta_1^2)\, d\bar{x} - \int_0^1 (-c_1\theta_u + c_2\theta_u^2)\, d\bar{x}. \tag{8.44}$$

Substituting instead of θ_1 and θ_u their values expressed through the angles α and θ', we obtain

$$c_y = \int_0^1 [c_1(\alpha+\theta_1')+c_2(\alpha^2+2\theta_1'\alpha+\theta_1'^2)]\,d\bar{x} -$$
$$-\int_0^1 [-c_1(\alpha+\theta_u')+c_2(\alpha^2+2\theta_u'\alpha+\theta_u'^2)]\,d\bar{x},$$

or

$$c_y=\alpha c_1+c_1\int_0^1\theta_1'd\bar{x}+c_2\alpha^2+2c_2\alpha\int_0^1\theta_1'd\bar{x}+c_2\int_0^1\theta_1'^2d\bar{x}+$$
$$+c_1\alpha+c_1\int_0^1\theta_u'd\bar{x}-c_2\alpha^2-2c_2\alpha\int_0^1\theta_u'd\bar{x}-c_2\int_0^1\theta_u'^2dx.$$

Grouping the terms, we find /199

$$c_y=c_1\int_0^1(\theta_1'+\theta_u')\,d\bar{x}+c_2\int_0^1(\theta_1'^2-\theta_u'^2)\,d\bar{x}+\alpha\left[2c_1+2c_2\int_0^1(\theta_1'-\theta_u')\,d\bar{x}\right]. \qquad (8.45)$$

Calculating as well for the values X, we will have

$$c_x=c_1\int_0^1(\theta_1'^2+\theta_u'^2)\,d\bar{x}+c_2\int_0^1(\theta_1'^3-\theta_u'^3)\,d\bar{x}+$$
$$+\left[2c_1\int_0^1(\theta_1'+\theta_u')\,d\bar{x}+3c_2\int_0^1(\theta_1'^2-\theta_u'^2)\,d\bar{x}\right]\alpha+$$
$$+\left[2c_1+3c_2\int_0^1(\theta_1'-\theta_u')\,d\bar{x}\right]\alpha^2. \qquad (8.46)$$

Similarly we may obtain the expression for the coefficient of the moment:

$$c_m=-\left[c_1\int_0^1(\theta_1'+\theta_u')\,\bar{x}d\bar{x}+c_2\int_0^1(\theta_1'^2-\theta_u'^2)\,\bar{x}d\bar{x}\right]-$$
$$-\left[c_1+2c_2\int_0^1(\theta_1'-\theta_u')\,\bar{x}d\bar{x}\right]\alpha. \qquad (8.47)$$

In expressions (8.45) to (8.47), for profiles with sharp edges

$$\int_0^1 \theta_l' d\bar{x} = \int_0^1 \frac{dy_l}{dx} d\bar{x} = 0,$$

$$\int_0^1 \theta_u' d\bar{x} = \int_0^1 \frac{dy_u}{dx} d\bar{x} = 0.$$

The integrals $\int_0^1 \theta_l'^3 d\bar{x}$ and $\int_0^1 \theta_u'^3 d\bar{x}$ are small values which may be ignored in the second approximation theory. In such a case, formulas (8.45) to (8.47) /200 may be rewritten in the following form

$$c_y = 2c_1\alpha + Ac_2, \tag{8.48}$$

$$c_x = Bc_1 + 3Ac_2\alpha + 2c_1\alpha^2, \tag{8.49}$$

$$c_m = -(c_1 + Dc_2)\alpha - (Ec_1 + Fc_2), \tag{8.50}$$

where

$$\left.\begin{aligned} A &= \int_0^1 (\theta_l'^3 - \theta_u'^3)\, d\bar{x} \\ B &= \int_0^1 (\theta_l'^2 + \theta_u'^2)\, d\bar{x} \\ D &= 2\int_0^1 (\theta_l' - \theta_u')\bar{x} d\bar{x} \\ E &= \int_0^1 (\theta_l' + \theta_u')\bar{x} d\bar{x} \\ F &= \int_0^1 (\theta_l'^2 - \theta_u'^2)\bar{x} d\bar{x} \end{aligned}\right\} \tag{8.51}$$

It is easy to see that if in the expressions we obtain the coefficient c_2 is considered equal to zero, we obtain formulas for linear theory.

From formulas (8.48) to (8.51) it follows that the effect of the Mach number on the aerodynamic characteristics of the profile is calculated only by the coefficients c_1 and c_2. The dependence of these coefficients on the Mach number, calculated according to formula (8.40), is shown in Figure 8.15.

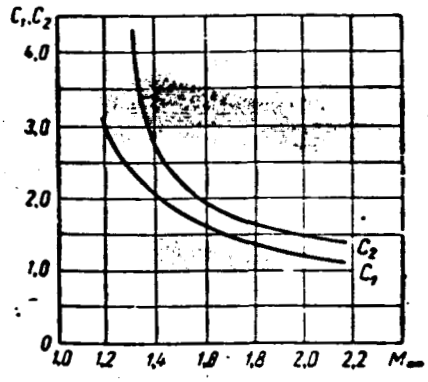


Figure 8.15. The Dependence of the Coefficients c_1 and c_2 on the Mach Number.

The effect of the shape of the profile, however, is wholly determined by the values of the parameters A, E, D, E, F (8.51). These parameters are determined by the geometrical parameters of the profile and for a plane plate are equal to zero. Of the five parameters, three - A, E and F - determine the asymmetry of the profile (for a symmetrical profile they are equal to zero). The parameters B and D are functions of the thickness of the profile, in which $B > 0$ while $D < 0$.

Studies conducted in the S.Ordzhonikidze Aviation Institute in Moscow by A. A. Lebedev established the following dependence of the aerodynamic characteristics of a profile in a supersonic flow on its structural parameters /201

$$c_y = 2c_1(\alpha - \alpha_0), \tag{8.52}$$

$$\alpha_0 = K_1 \bar{c}\bar{f}\frac{c_2}{c_1},$$

$$c_x = K_1\left(\frac{\bar{c}^3}{2} + \bar{f}^2\right)c_1 + \left(\alpha - \frac{3}{2}\alpha_0\right)^2 2c_1, \tag{8.53}$$

$$\bar{x}_F = \frac{1}{2}\left(1 - K_2\bar{c}\frac{c_2}{c_1}\right), \tag{8.54}$$

$$c_{m_0} = \left(-\bar{x}_F K_1 2\bar{c}\bar{f} + K_3\bar{c}\bar{f}\right)c_2 - K_2 c_1\bar{f}, \tag{8.55}$$

where α_0 is the angle of attack of the profile for $c_y = 0$;

$\bar{c}$ is the relative thickness of the profile;

$\bar{f}$ is the relative concavity of the profile;

$\bar{x}_F$ is the relative coordinate of the profile center;

c_{m_0} is the coefficient of the moment for $c_y = 0$;

K_1, K_2, K_3 are several numerical coefficients which are functions of the shape of the profile.

The coefficient K_1 coincides with the corresponding coefficient in linear theory (Table 8.1), while the values of the coefficients K_2 and K_3 for several profiles are given in Table 8.2.

From formula (8.53) it follows that c_{xu} for $c_y = 0$ is proportional to the square of the relative thickness and the relative concavity of the profile.

The theory examined earlier for a profile in a supersonic flow is a second approximation theory. This theory, developed by A. Ye. Donov, is a more

precise theory.

TABLE 8.2

Profile	Rhomboid	Rectangular	Formed by arcs of a circle
K_2	1	1	4/3
K_3	4	$2/1-\bar{x}_c$	16/3

A. E. Donov obtained the formula for the aerodynamic coefficients c_x and c_y with consideration for the fourth degree of the angle θ. In conjunction with the fact that his results do not clearly express the dependence of the aerodynamic characteristics for the profile on its geometric parameters, they are not introduced here.

§ 8.6. A Rigorous Solution to the Problem of Supersonic Flow Around Profiles Consisting of Rectalinear Segments. /202

The precise value for the pressure on the surface of a plane plate AB in a supersonic flow may be found in accordance with the theory of oblique shock waves (for the lower surface) and the results of the Prandtl-Mayer flow (for the upper surface). For given values M_∞ and α (Figure 8.16), using the graph for the dependence of the angle β on the angle of shift of flow α (Figure 5.3), we must determine the angle of incidence of the shock wave β. Then the ratio of pressure on the lower surface p_1 and pressure before the shock wave p_∞ may be found according to formula (5.6).

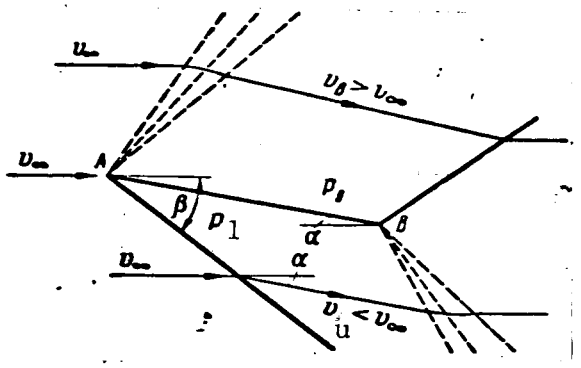


Figure 8.16. Rigorous Determination of Pressure on the Surface of a Plane Plate.

There is rarefied flow on the upper surface. Using the results of the Prandtl-Mayer flow (§ 6.8), we may determine the fictitious angle of shift θ_ϕ for the sonic flow before reaching the given Mach value or pressure p_∞ equal to

$$\frac{p_\infty}{p_0} = (1+0{,}2M_\infty^2)^{-3.5}.$$

To do so, we may use the table compiled according to the results of the Prandtl-Mayer flow, or the curve shown in Figure 6.17. The summary angle of shift

for sonic flow at a given angle of attack α will equal $\theta_\phi + \alpha$. According to the value of this angle, we may determine the ratio of pressure on the upper surface to the pressure of isentropic friction $\frac{p_u}{p_0}$ (Figure 6.17), while

$$\frac{p_u}{p_\infty} = \frac{p_u}{p_0}\frac{p_0}{p_\infty}$$

According to the value of the ratios of pressures $\frac{p_1}{p_\infty}$ and $\frac{p_u}{p_\infty}$, we may determine the pressure coefficients /203

$$\left.\begin{aligned} \bar{p}_1 &= \frac{2}{\kappa}\frac{1}{M_\infty^2}\left(\frac{p_1}{p_\infty} - 1\right) \\ \bar{p}_u &= -\frac{2}{\kappa}\frac{1}{M_\infty^2}\left(1 - \frac{p_u}{p_\infty}\right) \end{aligned}\right\}. \qquad (8.56)$$

Figure 8.17 shows the comparison of the actual pressure distribution along the plane plate with results of linear theory. From the figure it is clear that the linear theory yields decreased pressure both for the lower and upper surfaces. As was noted in § 8.2, this is explained by the fact that in linearization a shock of finite intensity on the lower surface is replaced by a shock of infinitely small intensity with an angle $\beta = \mu$, while on the upper surface the tensile of rarefaction lines is replaced by a single rarefaction line. In connection with this, the error in determining the total aerodynamic coefficients according to the formulas of linear theory is less than in determining the pressure.

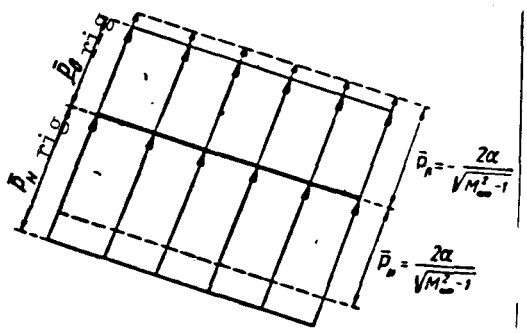


Figure 8.17. Comparison of the True Pressure Distribution Along a Plane Plate with Results of Linear Theory.

Similarly we compute the pressure on separate sections of the profile consisting of rectangular segments (Figure 8.18).

For flow around the lower surface, on the leading edge there appears an oblique shock wave corresponding to the angle of shift of the flow equal to $\alpha + \delta_1$. For flow around the upper surface the shock wave appears if $\alpha < \delta_u$. When this is true, the angle of shift of the flow equals $\delta_u - \alpha$ (Figure 8.18a). According to the value of the Mach number and the value of the angles $\alpha + \delta_1$ and $\delta_u - \alpha$, and using the relationships for an oblique shock wave, we may determine on the segments AB and AB_1 all the required parameters for the flow: /204

velocity, pressure, density, temperature, friction pressure, etc.

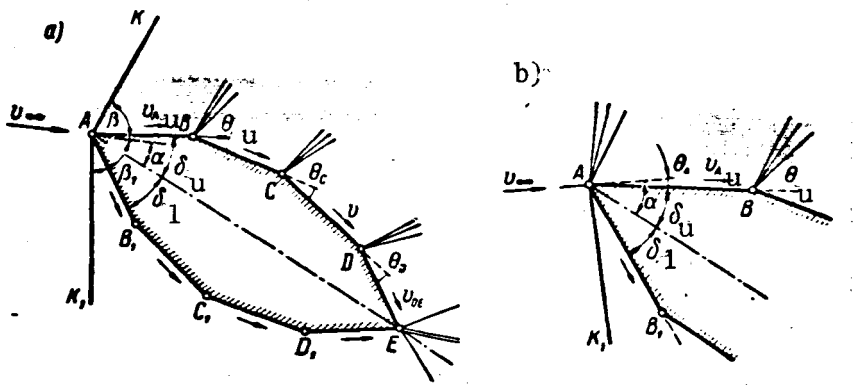


Figure 8.18. Supersonic Flow Around Profiles Consisting of Rectilinear Segments.

Between the leading and trailing edges on both the upper and lower surfaces there appears rarefied flow -- the successive flow of a series of angles exceeding 180°: ABC, BCD, CDE (on the upper surface) and $A_1B_1C_1$, $B_1C_1D_1$, $C_1D_1E_1$ (on the lower surface). The angle of shift of the flow in this case equal θ_B, θ_C, θ_D respectively.

Knowing the parameters of the flow behind the oblique shock waves (in segments AB and A_1B_1 (in Figure 8.18a) and using them as parameters for a nonturbulent flow, and using in addition the results of the Prandtl-Mayer flow, let us find all the parameters for the flow and the segments BC and B_1C_1. We may similarly calculate the velocity, pressure, density, etc., on the segments CD and C_1D_1. To do so we must use the parameters of the flow in the preceding segments BC and B_1C_1 as the parameters of a nonturbulent flow. Thus, we may in turn find the pressure over the entire surface of the profile.

For large angles of attack, when $\alpha > \delta_u$, in the region of the leading edge on the upper surface we encounter rarefied flow (Figure 8.18b). Therefore in determining the parameters of the flow from the segment AB we must use the results of the Prandtl-Mayer flow. This being so, the angle of shift in the flow equals $\alpha - \delta_u$.

We must emphasize that the method for calculating which we have just elaborated is applicable only for angles of attack α at which the angle of shift for the flow at the leading edge is less than the limiting angle for the given Mach number. For the opposite case, instead of an oblique shock we encounter a detached curvilinear shock wave.

PROBLEMS

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1. Using linear theory, determine the value of the components of the turbulence velocity related to the velocity of the nonturbulent flow $\frac{v'_x}{v_1}, \frac{v_y}{v_1}$ for flow around an oblique angle exceeding 180° if the angle of shift of the flow $\Delta\theta = 8°$ and $M_1 = 4$ are known. Compare the order of the values v'_x and v'_y.

2. Having fulfilled the conditions of problem 1, determine the relative pressure increase

$$\frac{\Delta p}{p_1} = \frac{p_2 - p_1}{p_1}.$$

3. Using the results of linear theory, find the values for the pressure coefficient on the upper and lower surfaces of a plane plate for $\alpha = 6°$ and $M_\infty = 4$. In addition, determine the values for $\bar{p}_l$ and $\bar{p}_u$ and the angle of inclination of the shock wave β on the lower surface, using the rigorous ratios. Compare the results obtained.

4. Construct the pressure distribution along the surface of a Rhomboid, rectangular and hexagonal profile for $\alpha = 0$ and for $M_\infty = 3$ and $\bar{c} = 0.04$. Construct the pressure distribution along the same profiles for $\alpha = 4°$.

5. Write the formula for determining the coefficients of maximum wave resistance for a rectangular profile given relative to the thickness for various values of the coordinate $\bar{x}_c$, determining the position of the maximum thickness and calculated from the leading edge of the profile.

6. Determine the ratio of the coefficients of minimum resistance in rectangular and Rhomboid profiles for various values of the relative coordinates for the maximum thickness $\bar{x}_c$ for a rectangular profile.

7. You are given the Mach numbers $M_\infty = 2, 3, 4$ and the angle of attack $\alpha = 4°$. Determine the ratio of the coefficient of lift to that of the wave resistance for Rhomboid, rectangular ($\bar{x}_c = 0.6$) and hexagonal ($\bar{x}_c = 0.3$) profiles for $\bar{c} = 0.04$. Compare the results obtained.

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